AP Calc AB
Sec 6.1 day 1 Differential Equations

Read p.404-405

We will cover goal #1 today: Use initial conditions to find particular solutions of differential equations.

Note: A Differential Equation is simply an equation with a derivative in it. A solution of the differential equation is not a number, but an equation or function.

A function is a solution of a differential equation if the equation is satisfied, meaning it is true, when y and its derivatives are substituted into the equation.

$Ex1$: Show that $y = e^{-2x}$ is a solution of the differential equation $y' + 2y = 0$.

First find all of the derivatives that are used in the differential equation.
if $y = e^{-2x}$, then $y' = -2e^{-2x}$. Substitute both of these into the differential eq.

$$y' + 2y = 0$$
$$-2e^{-2x} + 2(e^{-2x}) = 0$$
$$0 = 0$$

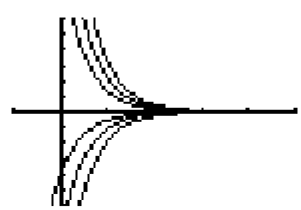
What if $y = 5e^{-2x}$ instead? $y' = 5(-2)e^{-2x}$ or $y' = -10e^{-2x}$.

$$y' + 2y = 0$$
$$-10e^{-2x} + 2(5e^{-2x}) = 0$$
$$0 = 0$$

In conclusion, every solution of $y' + 2y = 0$ is in the form $y = Ce^{-2x}$
where C is an y real number. $y = Ce^{-2x}$ is called the *general solution.*

Ex 1 involves a "1^{st} order" differential equation because y' is the highest-order derivative in the equation and the general solution involves 1 constant, C. A "2^{nd} order" differential equation involves y" and possibly y' and y and its general solution involves 2 arbitrary constants. It can be shown that a differential equation of the "nth order" has a general solution involving n arbitrary constants.

There are more examples on p.404.



If you choose values for the arbitrary constants and graph them, you will create a family of curves known as *solution curves.*
For ex1, the general solution is $y=Ce^{2x}$. Choosing arbitrary values for C gives us a family of curves like $y=12e^{2x}$, $y=5e^{2x}$, $y=8e^{2x}$, $y=-2e^{2x}$, $y=-4e^{2x}$, $y=-7e^{2x}$.

A **particular solution** can be created by simply substituting given values into the general solution. Recall that the given values are called the **initial conditions.**

$Ex2$: Verify the general solution, $y = C_1 + C_2 \ln x$ satisfies the differential equation, $xy'' + y' = 0$. Then find the particular solution that satisfies the initial condition $y=0$ when $x=2$ and $y' = \frac{1}{2}$ when $x=2$.

$$\text{if } y = C_1 + C_2 \ln x, \text{ then } y' = C_2 \frac{1}{x} = C_2 x^{-1}$$

$$\text{and } y'' = -C_2 x^{-2} \text{ or } y'' = \frac{-C_2}{x^2}$$

Therefore, $xy'' + y' = 0$

$$x(-C_2 x^{-2}) + C_2 x^{-1} = 0$$

$$-C_2 x^{-1} + C_2 x^{-1} = 0$$

$$0 = 0$$

The particular solution is found by the initial conditions.

Using $y' = \frac{1}{2}$ when $x=2$ and $y' = C_2 \frac{1}{x}$ we can find C_2.

$$\frac{1}{2} = C_2 \frac{1}{2} \text{ then } C_2 = 1$$

Also using $y=0$ when $x=2$ and $y = C_1 + C_2 \ln x$ with $C_2 = 1$ we can find C_1.

$$0 = C_1 + 1\ln 2$$

$$-1\ln 2 = C_1$$

The particular solution is $y = -1\ln 2 + \ln x$ or $y = \ln x - \ln 2 = \ln \frac{x}{2}$.

Homework is Sec 6.1 day 1 p.409-410 #1, 5, 9 – 27odd, 31, 33, 35, 37, 39, 42, 45, 47, 81.