Calc AB Notes 5.3 **Name__________________**

Inverse Functions:

f^{-1} the inverse function is formed by interchanging the x and y coordinates of a function.

Characteristics of the inverse function:

- If g is the inverse of f, then f is the inverse function of g.
- The domain of f^{-1} is equal to the range of f, and the range of f^{-1} is equal to the domain of f
- Not all functions have inverses.
- The graph of f^{-1} is a reflection of the graph of f in the line y = x. .
- Graphs of inverse functions have reciprocal slopes.

The definition:

A function g is the inverse function of the function f if :

$f(g(x)) = x$ for each x in the domain of g &

$g(f(x)) = x$ for each x in the domain of f .

Ex1: Show that the functions are inverse function of each other.

$$f(x) = 2x^3 - 1 \qquad and \qquad g(x) = \sqrt[3]{\frac{x+1}{2}}$$

Theorem 5.7—The existence of an Inverse Function

- A function has an inverse function if and only if it is one-to-one.
- If f is strictly monotonic on its entire domain then it is one-to-one and therefore has an inverse function (*passes the horizontal line test*)

Monotonic: *A function that it either increasing on its entire domain or decreasing on its entire domain.*

Ex2: Which function has an inverse function

A) $f(x) = 2x^3 - x - 1$ B) $f(x) = 2x^3 + x - 1$

Ex3: Find the inverse function: $f(x) = \sqrt{3x-4}$

Continuity and Differentiability of Inverse Functions:

Let f be a function whose domain is an interval I. If f has an inverse function, then the following are true.

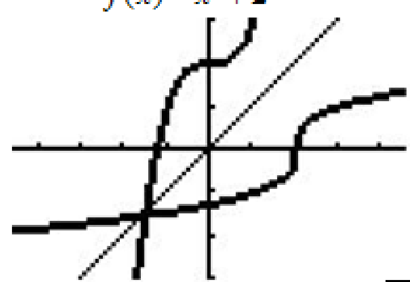
1. If f is continuous on its domain, then f^{-1} is continuous on its domain.
2. If f is increasing on its domain, then f^{-1} is increasing on its domain.
3. If f is decreasing on its domain, then f^{-1} is decreasing on its domain.
4. If f is differentiable at c and $f'(c) \neq 0$, then f^{-1} is differentiable at $f(c)$

Ex4:

a. Find f '(x) and g '(x).

b. Find f(1) and g(3).

c. Find f '(1) and g '(3)



The Derivative of an Inverse Function:

Let f be a function that is differentiable on an interval I. If f has an inverse function g, then g is differentiable at any x for which $f'(g(x)) \neq 0$ **AND**:

$$g'(x) = \frac{1}{f'(g(x))}, \qquad f'(g(x)) \neq 0$$

$Ex5$: Find $(h^{-1})'(3)$ given $h(x) = x^5 + 3x + 2$

$Ex6$: Find $(k^{-1})'(4)$ if $k(x) = 4x^3 + 2x - 5$