

Exponential Functions: Differentiation and Integration

The inverse of $f(x) = \ln x$ is $f^{-1}(x) = e^x$

Facts/Properties of Natural Exponential Functions:

1. $\ln(e^x) = x$ and $e^{\ln x} = x$
2. Product rule: $e^a e^b = e^{a+b}$
3. Quotient rule: $\frac{e^a}{e^b} = e^{a-b}$
4. Domain of e^x is all reals.
Range of e^x is positive reals.
5. e^x is continuous, increasing and 1:1 on its domain.
6. The graph of e^x is concave up on its domain.
7. $\lim_{x \rightarrow -\infty} e^x = 0$ and $\lim_{x \rightarrow \infty} e^x = \infty$.

Exponential/Log Review: Solve for x.

Ex1) $200e^{-4x} = 15$

Ex2) $\ln(x - 2) = 12$

Derivative of Exponential Functions:

$$1. \frac{d}{dx} e^x = e^x$$

$$2. \frac{d}{dx} e^u = e^u \frac{du}{dx}$$

Prove e^x is its own derivative:

Use $\ln e^x = x$

Find the derivative:

$$\text{Ex3) } y = x^2 e^{-x}$$

$$\text{Ex4) } y = \ln\left(\frac{1+e^x}{1-e^x}\right)$$

Integration Rules for Exponential Functions:

$$1. \int e^x dx =$$

$$2. \int e^u du =$$

$$\text{Ex5) } \int \frac{e^{2x}}{1+e^{2x}} dx$$

$$\text{Ex6) } \int_{\pi/3}^{\pi/2} e^{\sec 2x} \sec 2x \tan 2x dx$$