Calc Notes Name_____

DERIVATIVES OF BASES OTHER THAN e

Section 5.5

Reminders:

Definition of Exponential Function to Base a:
if a is any positive real number, not 1, and x is any real number, then

Rules for Exponents:

1. $a^0 = 1$
2. $a^x a^y = a^{x+y}$
3. $\left(a^x\right)^y = a^{xy}$
4. $\frac{a^x}{a^y} = a^{x-y}$

Definition of Logarithmic Function of Base a:
If a is a positive real number, not 1, and x is any real number, then

Rules of Logs:

1. $\log_a 1 = 0$
2. $\log_a xy = \log_a x + \log_a y$
3. $\log_a x^n = n \cdot \log_a x$
4. $\log_a \frac{x}{y} = \log_a x - \log_a y$

Change of Base Rule:

$$\log_a b = \frac{\log_* b}{\log_* a}$$

where * refers to any base!

Properties of Inverse Functions

1. $y = a^x$ if and only if $x = \log_a y$
2. $a^{\log_a x} = x, \quad x > 0$
3. $\log_a a^x = x$, for all x

*Ex*1) Solve: $\log_b 125 = 3$

*Ex*2) Solve: $3\left(5^{x-1}\right) = 86$

Derivative of the Natural Exponential function:

$$\frac{d}{dx}[e^u] = e^u \frac{du}{dx}$$

$$\frac{d}{dx}[\ln u] = \frac{du}{u}$$

Derivative of Bases other than e:

1. $\frac{d}{dx}\left[a^x\right] = (\ln a)a^x$
2. $\frac{d}{dx}\left[a^u\right] = (\ln a)a^u \frac{du}{dx}$
3. $\frac{d}{dx}[\log_a x] = \frac{1}{(\ln a)x}$
4. $\frac{d}{dx}[\log_a u] = \frac{1}{(\ln a)u} \frac{du}{dx}$

*Ex*3) Find the derivative of $f(x)=\dfrac{3^{2t}}{t}$

*Ex*4) Find the derivative of $f(x)=5^{-x/2}\sin 2x$

*Ex*5) Find the derivative of $f(x)=\log_3 \dfrac{x\sqrt{x-1}}{2}$

Integration of bases other than e $\int a^u du=\left(\dfrac{1}{\ln a}\right)a^u+C$

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*Ex*6) $\int 5^{-3x}dx$

*Ex*7) $\int_{-\pi/2}^{\pi/2} 2^{\sin x}\cos x\,dx$