Calc Notes Section 5.6

DERIVATIVES of Inverse Trig Functions Name______________________

None of the six basic trigonometric functions has an inverse because they are periodic and are not one-to-one. However, if we place certain restrictions on their domains we can find their inverses.

Definitions of Inverse Trigonometric Functions

Function	Domain	Range
$y = \arcsin x$ *iff* $\sin y = x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \arccos x$ *iff* $\cos y = x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \arctan x$ *iff* $\tan y = x$	$-\infty \le x \le \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = \text{arc}\cot x$ *iff* $\cot y = x$	$-\infty \le x \le \infty$	$0 < y < \pi$
$y = \text{arc}\sec x$ *iff* $\sec y = x$	$\lvert x \rvert \ge 1$	$0 \le y \le \pi, \quad y \ne \frac{\pi}{2}$
$y = \text{arc}\csc x$ *iff* $\csc y = x$	$\lvert x \rvert \ge 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}, \quad y \ne 0$

Evaluate each function:

Ex1: A) $\arcsin\left(\frac{-1}{2}\right) =$

B) $\arccos 0 =$

C) $\arctan \sqrt{3} =$

D) $\csc[\arctan\left(\frac{-5}{12}\right)] =$

Ex2: Write the expression in algebraic form:

A) $\sec(\arctan 4x)$

B) $\cos(\arcsin \frac{x-h}{r})$

Ex3: Solve the equation: $\arctan(2x-3) = \frac{\pi}{4}$

Derivatives of Inverse Trigonometric Functions

$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$	$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$	$\frac{d}{dx}[\text{arc}\sec u] = \frac{u'}{\lvert u \rvert \sqrt{u^2-1}}$
$\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$	$\frac{d}{dx}[\text{arc}\cot u] = \frac{-u'}{1+u^2}$	$\frac{d}{dx}[\text{arc}\csc u] = \frac{-u'}{\lvert u \rvert \sqrt{u^2-1}}$

Ex4: Find the derivative:

A) $f(x) = \arcsin x^2$

B) $f(x) = \text{arc}\sec 2x$

C) $f(x) = x^2 \arctan x$

D) $f(x) = x \arctan 2x - \frac{1}{4}\ln(1+4x^2)$