Calc Notes: Section 5.7
Inverse Trigonometric Functions: Integration   Name________________________

Remember Derivatives of Inverse Trig Functions-----they fall into three pairs: the derivative of one function is the negative of the other.

$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$	$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$	$\frac{d}{dx}[\operatorname{arc}\sec u] = \frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$
$\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$	$\frac{d}{dx}[\operatorname{arc}\cot u] = \frac{-u'}{1+u^2}$	$\frac{d}{dx}[\operatorname{arc}\csc u] = \frac{-u'}{\lvert u\rvert\sqrt{u^2-1}}$

So when we are taking the integral we only need to look at three integral types.

1. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right) + c$

2. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\operatorname{arc}\tan\left(\frac{u}{a}\right) + c$

3. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arc}\sec\frac{|u|}{a} + c$

Evaluate each Integral:

$Ex1)$ $\int \frac{\mathrm{dx}}{\sqrt{9\text{-}4\mathrm{x}^2}}$

$Ex2)$ $\int \frac{\mathrm{dx}}{5+16\mathrm{x}^2}$

$Ex3)$ $\int \frac{\mathrm{dx}}{\mathrm{x}\sqrt{9\mathrm{x}^2-1}}$

The following examples are less obvious. The integration formulas for inverse trig functions can be disguised in many ways.

$$Ex4)\ \int \frac{dx}{\sqrt{e^{2x}-4}}$$

Rewriting a quotient into a sum of 2 quotients:

$$Ex5)\ \int \frac{\text{x+3}}{\sqrt{\text{25-x}^2}}\,dx$$

Completing the Square:

$$\text{EX6)}\ \int \frac{\text{dx}}{\text{x}^2-6x+10}$$