

HONORS GEOMETRY—Summer Skills Set

Algebra Concepts

Adding and Subtracting Rational Numbers

To add or subtract fractions with the same denominator, add or subtract the numerators and write the sum or difference over the denominator. Simplify, if necessary.

Example: $\frac{3}{5} + \frac{1}{5} = \frac{3+1}{5} = \frac{4}{5}$

Example: $\frac{7}{16} - \frac{3}{16} = \frac{7-3}{16} = \frac{4}{16} = \frac{1}{4}$

To add or subtract fractions with different denominators, first find the least common denominator (LCD). Rewrite each fraction with the LCD, and then add or subtract. Simplify, if necessary.

Example: $\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6}$ (improper fraction) or $1\frac{1}{6}$ (mixed number)

Find each sum or difference. Write in simplest form.

1. $\frac{5}{16} - \frac{4}{16}$

2. $\frac{2}{15} + \frac{7}{15}$

3. $\frac{2}{3} + \frac{1}{3}$

4. $\frac{5}{4} - \frac{3}{4}$

5. $\frac{1}{2} - \frac{1}{3}$

6. $\frac{8}{9} - \frac{2}{3}$

7. $\frac{3}{7} + \frac{5}{14}$

8. $\frac{11}{12} - \frac{4}{15}$

9. $\frac{2}{5} + \frac{3}{4}$

10. $1 - \frac{1}{19}$

11. $\frac{9}{10} - \left(-\frac{3}{5}\right)$

12. $-\frac{2}{3} + \frac{5}{9}$

13. $\left(\frac{3}{4}\right) - \left(\frac{2}{7}\right)$

14. $\left(-\frac{4}{5}\right) - \left(\frac{1}{4}\right)$

15. $\left(-\frac{2}{5}\right) - \left(-\frac{1}{3}\right)$

1.	$\frac{1}{16}$
2.	$\frac{3}{5}$
3.	1
4.	$\frac{1}{2}$
5.	$\frac{1}{6}$
6.	$\frac{2}{9}$
7.	$\frac{11}{14}$
8.	$\frac{13}{20}$
9.	$\frac{23}{20}$ or $1\frac{3}{20}$
10.	$\frac{18}{19}$
11.	$\frac{3}{2}$ or $1\frac{1}{2}$
12.	$-\frac{1}{9}$
13.	$\frac{13}{28}$
14.	$-\frac{21}{20}$ or $-1\frac{1}{20}$
15.	$-\frac{1}{15}$

Algebra Concepts

KEY

Order of Operations

1. Evaluate within grouping symbols
 2. Evaluate powers (exponents)
 3. Multiply and divide in order (L→R)
 4. Add and subtract in order (L→R)
 5. Simplify as needed
- * A number next to a grouping symbol means multiply.

Examples:

$$\begin{aligned}
 & 3^2(5-3)^3+3 & 4+12\times 3-8\div 4 \\
 & = 3^2(2)^3+3 & = 4+36-2 \\
 & = 9(8)+3 & = 40-2 \\
 & = 72+3 & = 38 \\
 & = 75
 \end{aligned}$$

Evaluate each expression.

1. $6 - 5(7 - 5)^3 + 5$

2. $(4+5)-8+2(3)$

3. $(6-3)^2+12-8\div 2$

4. $36 \div 2(5-1)^2$

5. $-4+5(7-4)-(-3)\div 3$

6. $-7(8)+4(2)-(6+1)^2$

Evaluate each expression for $s = -3$ and $v = 2$

7. sv^2

8. $(sv)^2$

9. $-s^2+2s-4$

10. $s^2 - v^2$

11. $(s-v)^2$

12. $2s^2v$

1. -29

2. 7

3. 17

4. 288

5. 12

6. -97

7. -12

8. 36

9. -19

10. 5

11. 25

12. 36

- Factor out the greatest common factor (GCF) first
- Look for special cases next—the difference of two squares (two terms only) or a perfect square trinomial (three terms)
- Look for a pair of binomial factors (“reverse-FOIL”)
- If there are four or more terms, try grouping to find common binomial factors
- As a final check, be sure there are no common factors other than 1

Factor completely.

1. $x^2 + 9x + 18$

2. $x^2 - 6x + 9$

3. $x^2 - 5x - 24$

1. $(x+3)(x+6)$

4. $x^2 + 3x - 70$

5. $18x^2 + 57x + 24$

6. $x^4 - 10x^2 + 9$

2. $(x-3)^2$

3. $(x-8)(x+3)$

4. $(x+10)(x-7)$

5. $3(2x+1)(3x+8)$

6. $(x-1)(x+1)(x-3)(x+3)$

7. $-2p^2 + 28p - 66$

8. $16y^2 - 1$

9. $2x^2 - 9x - 5$

7. $-2(p-11)(p-3)$

8. $(4y-1)(4y+1)$

9. $(2x+1)(x-5)$

10. $(2x-1)(5x-2)$

10. $10x^2 - 9x + 2$

11. $20q^2 - 13q + 2$

12. $x^4 - x^2$

11. $(4g-1)(5g-2)$

12. $x^2(x+1)(x-1)$

13. $(3p^2+7)(4p-7)$

13. $12p^3 - 21p^2 + 28p - 49$

14. $6v^3 + 16v^2 + 21v + 56$

14. $(2v^2+7)(3v+8)$

The solutions of a quadratic equation are the x -intercepts of the graph of the corresponding parabola. There can be two real solutions, one real solution, or no real solution.

- First bring everything to one side (set the equation equal to 0).
- When there is no linear term ($b = 0$), get x^2 by itself and take the square root. Two answers result.
- If the quadratic expression can be factored easily, then factor, set each factor equal to zero and solve.
- When factoring is not easy or not possible, use the quadratic formula or solve by calculating the zeros on your graphing calculator.

The quadratic formula: If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solve each equation by the indicated method. When necessary, round answers to two decimal places.

Solve questions 1-3 by using square roots.

1. $k^2 = 16$

2. $x^2 + 7 = 25$

3. $2m^2 + 24 = 10$

1. $k = \pm 4$

2. $x \approx \pm 4.24$

3. no solution

4. $p = -6, 3$

5. $n = -5, -3$

6. $r = 1$

7. $g = -\frac{3}{2} \text{ or } -\frac{1}{2}$

8. $x = \pm 1.90$

9. $n \approx -0.38 \text{ or } 1.16$

10. $x = -3, 1$

11. $x \approx -1.57 \text{ or } 1.13$

12. $x \approx \pm 5.39$

Solve questions 4-6 by factoring.

4. $p^2 + 3p - 18 = 0$

5. $n^2 + 8n = -15$

6. $7r^2 - 14r = -7$

Solve questions 7-9 by using the quadratic formula. Check your answers by graphing.

7. $4g^2 + 8g + 7 = 4$

8. $5x^2 = 18$

9. $9n^2 - 7n - 4 = 0$

Solve questions 10-12 by an appropriate method.

10. $x^2 + 2x - 1 = 2$

11. $8x^2 + 4x - 16 = -x^2$

12. $10x^2 + 2 = 292$

Algebra Concepts

Solving Equations in One Variable

To solve an equation in one variable, use inverse operations to isolate the variable.

Example: $5 - 2(r + 6) = 1$

$$\begin{array}{ll} 5 - 2r - 12 = 1 & \text{Distributive Property} \\ -2r - 7 = 1 & \text{Combine like terms} \\ -2r - 7 + 7 = 1 + 7 & \text{Add 7 to each side} \\ -2r = 8 & \text{Simplify} \\ \frac{-2r}{-2} = \frac{8}{-2} & \text{Divide both sides by } -2 \\ r = -4 & \text{Simplify} \end{array}$$

Solve each equation.

1. $x - 6 = 10$

2. $\frac{x}{5} = 15$

3. $8x = 24$

1. $x = 16$

4. $-\frac{4}{7}x = -8$

5. $a - \frac{1}{8} = \frac{5}{8}$

6. $3y - 4 = 20$

2. $x = 75$

7. $\frac{t}{7} + 2 = 1$

8. $3r - (2r + 1) = 21$

9. $44 = 5y - 8 - y$

3. $x = 3$

10. $75 + 7c = 2c$

11. $\frac{3}{5}n + 12 = 2n - 9$

12. $-\frac{1}{2}(16 - 2y) = 11$

4. $x = 14$

13. $7(4c + 1) - 2(2c - 3) = -23$

14. $x - (-4x + 2) = 13$

5. $a = \frac{3}{4}$

6. $y = 8$

7. $t = -7$

8. $r = 22$

9. $y = 13$

10. $c = -15$

11. $n = 15$

12. $y = 19$

13. $c = -\frac{3}{2}$

14. $x = 3$

To solve a system of equations use the substitution method.

Example: Solve the linear system.

$$x + y = 1$$

Equation 1

$$2x - 3y = 12$$

Equation 2

SOLUTION

Solve for y in Equation 1.

$$y = -x + 1$$

Revised Equation 1

Substitute $-x + 1$ for y in Equation 2 and solve for x .

$$2x - 3y = 12$$

Write Equation 2.

$$2x - 3(-x + 1) = 12$$

Substitute $-x + 1$ for y .

$$2x + 3x - 3 = 12$$

Distribute the -3 .

$$5x - 3 = 12$$

Simplify.

$$5x = 15$$

Add 3 to each side.

$$x = 3$$

Solve for x .

To find the value of y , substitute 3 for x in the revised Equation 1.

$$y = -x + 1$$

Write revised Equation 1.

$$y = -3 + 1$$

Substitute 3 for x .

$$y = -2$$

Solve for y .

The solution is $(3, -2)$.

1. $4x + y = 9$

$$y = -7$$

2. $3x = 9$

$$-2x + y = -8$$

3. $x - 2y = -13$

$$y = -2x - 6$$

1. $(4, -7)$

4. $x - y = 10$

$$5x - y = -6$$

5. $4x + y = 2$

$$x - y = -17$$

6. $-x + 3y = 4$

$$x + 6y = 14$$

2. $(3, -2)$

3. $(-5, 4)$

4. $(-4, -14)$

5. $(-3, 14)$

6. $(2, 2)$

7. $(4, -2)$

8. $(2, 1)$

9. $(-3, 2)$

7. $3x + 2y = 8$

$$x + 4y = -4$$

8. $x - 5y = -3$

$$4x - 3y = 5$$

9. $2x + 5y = 4$

$$x + 5y = 7$$

To solve a system of equations using linear combinations.

Example:

Solve the linear system.

4x - 3y = 11	Equation 1
3x + 2y = -13	Equation 2

SOLUTION

The equations are arranged with like terms in columns. You can get the coefficients of y to be opposites by multiplying the first equation by 2 and the second equation by 3.

4x - 3y = 11	Multiply by 2.	8x - 6y = 22
3x + 2y = -13	Multiply by 3.	<u>9x + 6y = -39</u>
		17x = -17 Add the equations.
		x = -1 Solve for x.

Substitute -1 for x in the second equation and solve for y.

3x + 2y = -13	Write Equation 2.	
3(-1) + 2y = -13	Substitute -1 for x.	
-3 + 2y = -13	Simplify.	
y = -5	Solve for y.	

The solution is (-1, -5).

1. $x + y = 11$
 $x - y = 7$

2. $x - 2y = 8$
 $-x + 3y = -15$

3. $3x + y = -8$
 $-3x + 4y = -2$

4. $4x - 5y = -18$
 $5x + 4y = -2$

5. $2x + 5y = -22$
 $4x - 3y = 8$

6. $4x = -3 + y$
 $y = -6x - 7$

7. $x + 2y = -3$
 $x - 4y = 15$

8. $-x - 5y = 30$
 $2x - 7y = 25$

9. $-x + 8y = 16$
 $3x + 4y = 36$

- | |
|---------------|
| 1. $(9, 2)$ |
| 2. $(-6, -7)$ |
| 3. $(-2, -2)$ |
| 4. $(-2, 2)$ |
| 5. $(-1, -4)$ |
| 6. $(-1, -1)$ |
| 7. $(3, -3)$ |
| 8. $(-5, -5)$ |
| 9. $(8, 3)$ |

Score _____

Example:

Write an equation of the line that passes through the point $(-2, 5)$ and has a slope of 3.

SOLUTION

Find the y-intercept.

$$y = mx + b \quad \text{Write slope-intercept form.}$$

$$5 = 3(-2) + b \quad \text{Substitute 3 for } m, -2 \text{ for } x, \text{ and 5 for } y.$$

$$5 = -6 + b \quad \text{Simplify.}$$

$$11 = b \quad \text{Solve for } b.$$

The y-intercept is $b = 11$.

Now write an equation of the line, using slope-intercept form.

$$y = mx + b \quad \text{Write slope-intercept form.}$$

$$y = 3x + 11 \quad \text{Substitute 3 for } m \text{ and 11 for } b.$$

Write an equation of the line that passes through the point and has the given slope. Write the equation in slope-intercept form.

1. $(3, 5), m = -1$

2. $(-2, 6), m = 4$

3. $(7, -2), m = -3$

4. $(2, 8), m = 0$

5. $(-3, 0), m = 2$

6. $(0, 0), m = -7$

7. $(0, -2), m = -\frac{5}{3}$

8. $(-5, -1), m = \frac{3}{4}$

9. $(3, -2), m = -\frac{5}{7}$

1. $y = -x + 8$

2. $y = 4x + 14$

3. $y = -3x + 27$

4. $y = 8$

5. $y = 2x + 6$

6. $y = -7x$

7. $y = -\frac{5}{3}x - 2$

8. $y = \frac{3}{4}x + \frac{11}{4}$

9. $y = -\frac{5}{7}x + \frac{1}{7}$

Example:

Write an equation of the line that passes through the points $(1, 5)$ and $(2, 3)$.

SOLUTION

Find the slope of the line. Let $(x_1, y_1) = (1, 5)$ and $(x_2, y_2) = (2, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Write formula for slope.

$$= \frac{3 - 5}{2 - 1}$$

Substitute.

$$= \frac{-2}{1} = -2$$

Simplify.

Find the y -intercept. Let $m = -2$, $x = 1$, and $y = 5$ and solve for b .

$$y = mx + b$$

Write slope-intercept form.

$$5 = (-2)(1) + b$$

Substitute -2 for m , 1 for x , and 5 for y .

$$5 = -2 + b$$

Simplify.

$$7 = b$$

Solve for b .

Write an equation of the line.

$$y = mx + b$$

Write slope-intercept form.

$$y = -2x + 7$$

Substitute -2 for m and 7 for b .

Write an equation in slope-intercept form of the line that passes through the points.

1. $(5, 0), (-10, -5)$

2. $(1, 1), (3, 3)$

3. $(1, -7), (3, -15)$

4. $(-6, -2), (-10, -14)$

5. $(2, 3), (6, 11)$

6. $(0, 2), (-2, 0)$

7. $(0, 0), (3, -6)$

8. $(0, 4), (-1, 3)$

9. $(-5, 9), (-2, 0)$

- | |
|-------------------------------------|
| 1. $y = \frac{1}{3}x + \frac{5}{3}$ |
| 2. $y = x$ |
| 3. $y = -4x - 3$ |
| 4. $y = 3x + 16$ |
| 5. $y = 2x - 1$ |
| 6. $y = x + 2$ |
| 7. $y = -2x$ |
| 8. $y = x + 4$ |
| 9. $y = -3x - 6$ |

To simplify radicals, look for perfect squares that divide evenly into the radicand.

Perfect squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, et cetera...

$$\text{Example: } \sqrt{56} = \sqrt{4 \cdot 14} = \sqrt{4} \cdot \sqrt{14} = 2\sqrt{14}$$

Use the product or quotient rule to simplify radicals. Multiply rational parts separately.

$$\text{Example: } 2\sqrt{7} \cdot 3\sqrt{14} = 2 \cdot 3 \sqrt{7 \cdot 14} = 6\sqrt{7 \cdot 7 \cdot 2} = 6\sqrt{7^2} \cdot \sqrt{2} = 6 \cdot 7\sqrt{2} = 42\sqrt{2}$$

$$\text{Example: } \frac{\sqrt{6z^2w}}{\sqrt{2zw}} = \sqrt{\frac{6z^2w}{2zw}} = \sqrt{\frac{3z^2w}{z^2w}} = \sqrt{3z}$$

To add or subtract radicals, the radicand must be the same. Simplify individual radicals first.

$$\text{Example: } \sqrt{18} - \sqrt{8} + \sqrt{44} = 3\sqrt{2} - 2\sqrt{2} + 2\sqrt{11} = \sqrt{2} + 2\sqrt{11}$$

Simplify each expression. Write in simplest radical form.

1. $\sqrt{45}$

2. $\sqrt{216}$

3. $5\sqrt{128}$

4. $\sqrt{5} \cdot \sqrt{10}$

5. $3\sqrt{12} \cdot \sqrt{6}$

6. $\frac{\sqrt{72}}{\sqrt{8}}$

7. $\sqrt{\frac{16}{81}}$

8. $\frac{3\sqrt{20}}{2\sqrt{4}}$

9. $-3\sqrt{7} + 5\sqrt{7}$

10. $10\sqrt{15} - 9\sqrt{15}$

11. $-11\sqrt{21} - 11\sqrt{21}$

12. $-2\sqrt{3} + 3\sqrt{27}$

13. $2\sqrt{6} - 2\sqrt{24}$

14. $2\sqrt{45} - 2\sqrt{5} + \sqrt{20}$

1. $3\sqrt{5}$

2. $6\sqrt{6}$

3. $40\sqrt{2}$

4. $5\sqrt{2}$

5. $18\sqrt{2}$

6. 3

7. $\frac{4}{9}$

8. $\frac{3\sqrt{5}}{2}$

9. $2\sqrt{7}$

10. $\sqrt{15}$

11. $-22\sqrt{21}$

12. $7\sqrt{3}$

13. $-2\sqrt{6}$

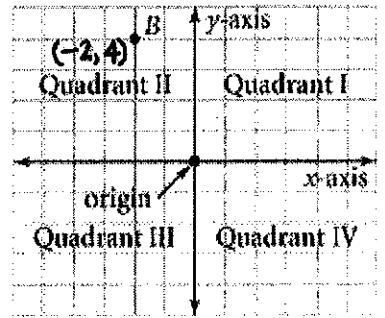
14. $6\sqrt{5}$

Ordered pairs can be graphed on a coordinate plane. The first number of an ordered pair shows how to move *across*. It is called the **x**-coordinate.

The second number of an ordered pair shows how to move *up or down*. It is called the **y**-coordinate.

Example:

To locate point *B*, move left (backward) to -2 and up to 4 .



Give the coordinates of the following labeled points.

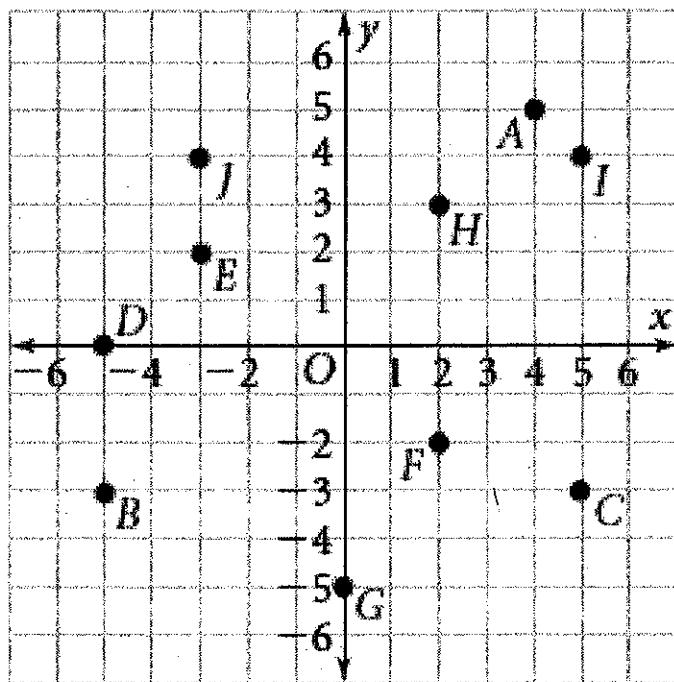
1. A

2. B

3. C

4. D

5. E



- | |
|-------------|
| 1. (4, 5) |
| 2. (-4, -3) |
| 3. (5, -3) |
| 4. (-4, 0) |
| 5. (-3, 2) |
| 6. J |
| 7. I |
| 8. G |
| 9. H |
| 10. F |

Match the coordinates to the corresponding point labeled on the above graph.

6. $(-3, 4)$ 7. $(5, 4)$ 8. $(0, -5)$ 9. $(2, 3)$ 10. $(2, -2)$