5.1 Exponential Functions and Their Graphs

$$\boxed{\text{answer} = \text{base}^{\text{exponent}}}$$

*exponential forward → given an exponent → find answer

*logarithm backward → given an answer → find exponent

Roll #	0	1	2	3	4	5					
# Students Standing	31	11	3	2	2	1					

$$y = \underbrace{20}_{\text{initial value}}(\underbrace{.52}_{\text{G/D factor}})^x$$

Exponential Functions

Exponential Function:

*Parent function

$$f(x)=a^x$$

$\text{answer} = \text{base}^{\text{exponent}}$

where $a>0$, $a\neq 1$, and x is any real number

pos.

$a>1$

$$\underbrace{\text{answer}}_{\text{pos.}} = \underbrace{\text{base}}_{\text{pos.}}{}^{\text{exponent}}$$

anything real!

Evaluate:

a) $f(x) = 2^x \quad x = -3.1$

.116..

b) $f(x) = 2^{-x} \quad x = \pi$

.113

c) $f(x) = .6^x \quad x = ½$

.7745

exponent can be anything!

answer will always be positive!

General Facts

$$f(x) = a^x$$

output → answer = base$^{\text{exponent}}$ ← input

-Domain: $(-\infty,\infty)$ all $\mathbb{R}$'s

-Range: $(0,\infty)$ → Horiz. Asymptote

-Intercept (0,1)

-Horizontal Asymptote at x-axis

-increasing if $a>1$ Growth

-decreasing if $0<a<1$ Decay

$\boxed{\text{* parent function}}$

How to sketch exponential

1. Find the y-intercept (make $x=0$)

2. Plot (1,a) (make $x=1$)

2.5 → Horiz. Asymp.

3. Plot points on both sides of intercept

Graph:

$y=2^x$

$y=2^{x+1}$ $\rightarrow$ left 1

$y=2^{x+1}-3$ $\rightarrow$ left 1
down 3
HA: $y=-3$

Shifts:

UP: $f(x) + c$

DOWN: $f(x) - c$

RIGHT: $f(x - c)$

LEFT: $f(x + c)$

Reflecting:

Across the x-axis: $-f(x)$

Across the y-axis: $f(-x)$

Transformations

$$f(x) = \pm a^{x \pm c} \pm b$$

zero? no!

$f(x) \overset{?}{=} \pm b$ No!

c: shifts graph horizontally

b: shifts graph vertically

-a: flips graph

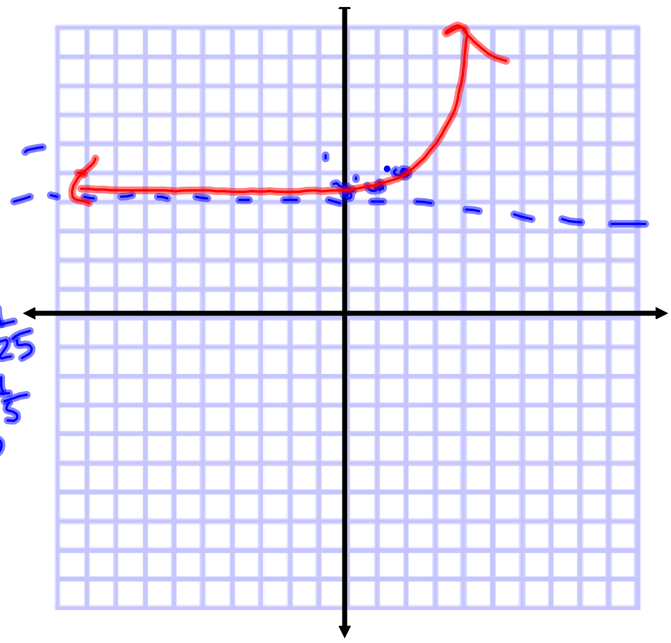
$b \rightarrow$ Horizontal asymptote

Graph:

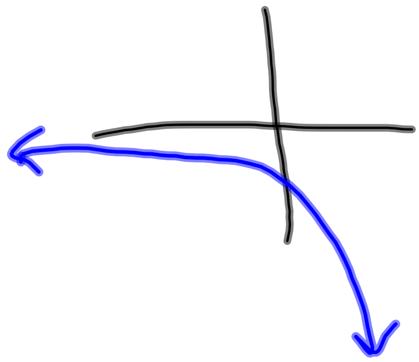
$f(x)=5^{x-2}+4$

right 2
up 4

x	y
0	$4\frac{1}{25}$
1	$4\frac{1}{5}$
2	5



$f(x) = -3^{x+1} -2$



One to One Property:

$2^2 = 2^?$ $4^3 = 4^?$

$? = 2$ $? = 3$

If $x^n = x^m$, then $n=m$

$9^{2x+1} = 81^{3x-2}$

$9^{2x+1} = (9^2)^{3x-2}$

$9^{2x+1} = 9^{6x-4}$

$2x+1 = 6x-4$

$x = \frac{5}{4}$

$6^{4x-3} = 36^{x+8}$

$5^x = 16^{2x}$

stay tuned....

$9^x = 27^{2x-1}$

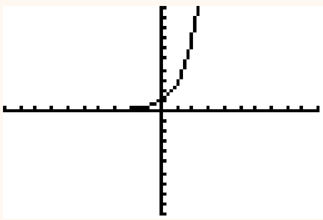
$3^{2x} = 3^{6x-3}$

Applications

The Natural Base e:

$$\left(1+\frac{1}{x}\right)^x \approx 2.718281828......$$

irrational



X	Y_1
-3	.04979
-2	.13534
-1	.36788
0	1
1	2.7183
2	7.3891
3	20.086

X= -3

Applications: Compound Interest (n compoundings per year)

Principal P - \$1000

Annual Interest Rate r - 15%

Compounded once a year

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

rate, time, initial principle, # of compound

Time in years	Balance after each compounding
0	$A =$ 1000
1	$A_1 = 1000\left(1+\frac{.15}{1}\right)^1$
2	$A_2 = 1000\left(1+\frac{.15}{1}\right)^2$
t	$A_t =$

- For more frequent compounding (quarterly, monthly, etc.) let n be the number of compoundings per year and let t be the number of years. Then the new account balance after t years is:

Applications: Continuous Compounding

Continuous Compounding:
When the number of compoundings n increase without bound.

Formula for continuous compounding:
$A = Pe^{rt}$

An investment of $5,000 is made into an account that pays 6% annual interest for 10 years. Find the amount in the account if the interest is compounded:

a) annually $n = 1$

$$= 5000\left(1 + \frac{.06}{1}\right)^{10}$$

$$=$$

b) quarterly

$n = 4$

$$= 5000\left(1 + \frac{.06}{4}\right)^{40}$$

c) monthly $n = 12$

$$= 5000\left(1 + \frac{.06}{12}\right)^{120}$$

d) daily $n = 365$

$$= 5000\left(1 + \frac{.06}{365}\right)^{3650}$$

e) continuously $\rightarrow e$

$$= 5000\, e^{(.06)(10)}$$