



## 5.2 Logarithms



"Okay, Bob! Go! Go!"

$$\frac{\text{answer}}{\text{pos.}} = \frac{\text{base}}{\text{pos.}} \frac{\text{exponent}}{\text{anything}}$$

exponential: given exponent, find answer

↓

logarithm: given answer, find exponent

**Solve:**

$$5^x = 25$$

$$x = 2$$

$$7^x = 178$$

$$\sqrt[x]{178}$$

$$x =$$

# Logarithmic Function

For  $x > 0$ ,  $a > 0$ , and  $a \neq 1$ ,

$y = \log_a x$ , if and only if  $x = a^y$

*exponent* →  $y = \log_a x$  *base* *answer*

exponent = log base answer

*output* *anything* base *pos.* answer *input* *pos.*

Logarithms are exponents!

$$\text{a) } \log_2 32 = 5$$
$$2^? = 32$$

$$\text{b) } \log_4 2 = \frac{1}{2}$$
$$4^? = 2$$

$$\text{c) } \log_3 1 = 0$$
$$3^? = 1$$

$$\text{d) } \log_{10} (1/100) = -2$$

$$10^? = \frac{1}{100}$$

Common Logarithm: a logarithm function with base 10

$$\text{Log}_{10} 10 = 1$$

$10^? = 10$

$$\text{Log } 1/4 =$$

$10^? = \frac{1}{4}$

$$\log(.25) = -.6020599913$$

Natural Log: a logarithm function with base  $e$

$$\ln e = 1$$

$$e^1 = e$$

$$\ln 58 =$$

$$e^? = 58$$

$$\ln(58) = 4.060443011$$



## Properties of Logarithms

1.  $\log_a 1 = 0$

$$a^0 = 1$$

2.  $\log_a a = 1$

$$a^1 = a$$

3.  ~~$\log_a a^x = x$~~

$$a^? = a^x$$



$$\sqrt{x^2} = x$$

$$(\sqrt{x})^2 = x$$

4. If  $\log_a x = \log_a y$  then  $x = y$

one to one

\* Drop logs

a) Solve for  $x$ :  $\log_2 x = \log_2 3$

$x = 3$

b) Solve for  $x$ :  $\log_4 4 = x$

$4^x = 4 \quad x = 1$

c) **Simplify**:  $\log_5 5^x$

~~$x$~~

$5^? = 5^x$

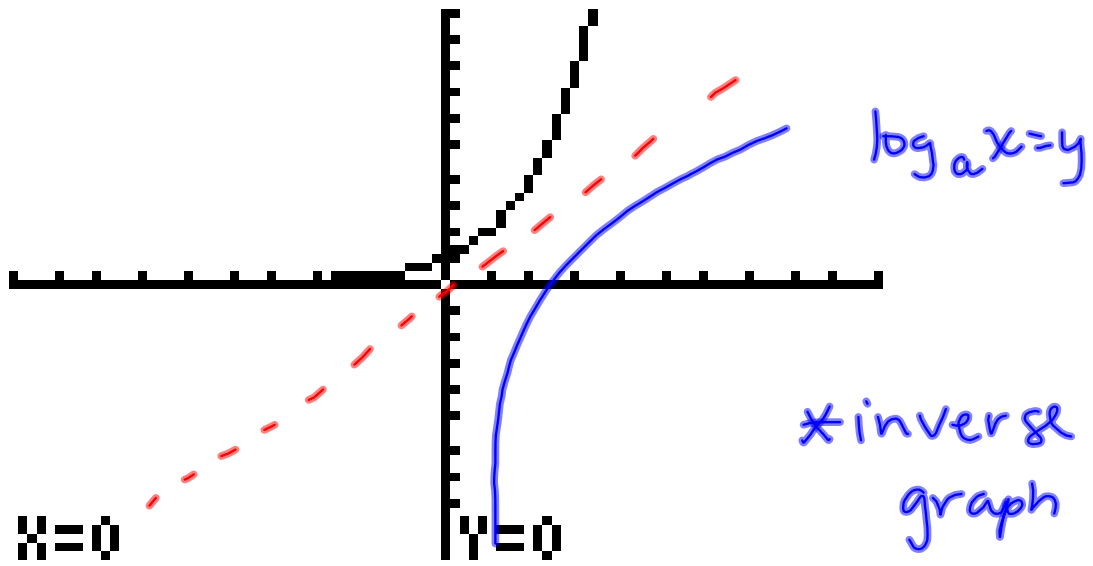
d) Simplify:  ~~$7 \log_7 14$~~

$14$

$\sqrt{x^2} = x$

$(\sqrt{x})^2 = x$

$$y = a^x$$

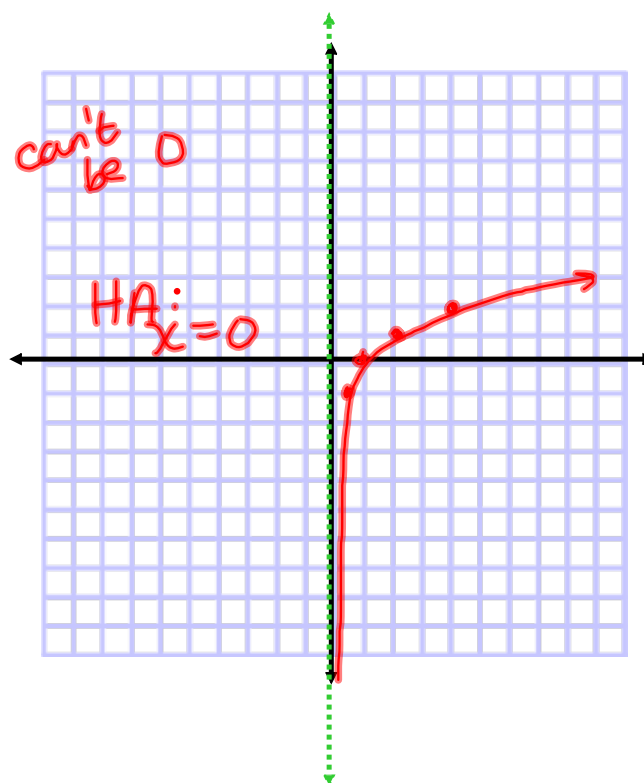


Graph:

$$f(x) = \log_2 x$$

$$2^? = x$$

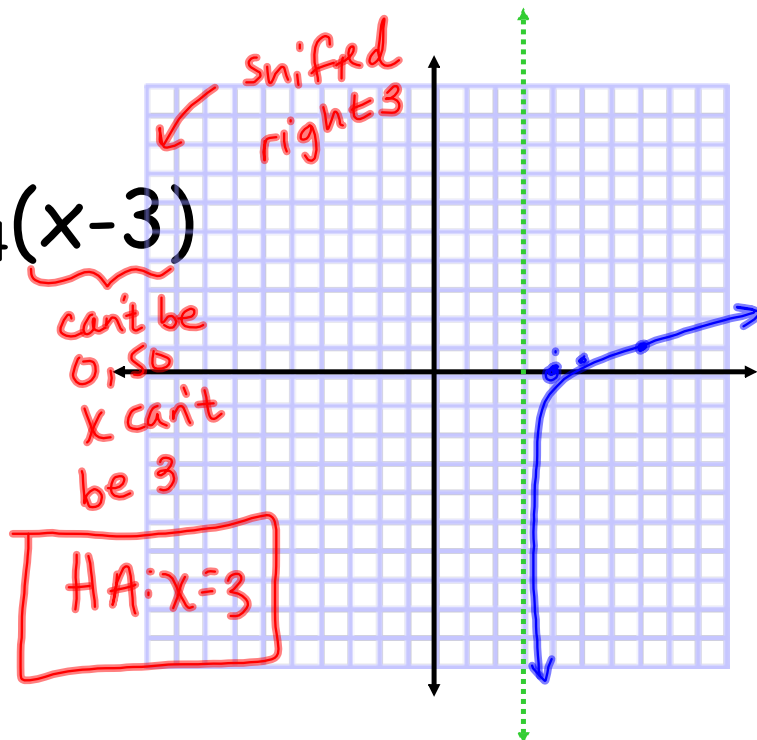
$x$	
1	0
2	1
4	.2
$\frac{1}{2}$	-1



Graph:

$$f(x) = \log_4(x-3)$$

$x$	$4^?$
4	0
5	$\frac{1}{2}$
7	1
19	2



Graph:  $h(x) = -\log_6(x+2)$

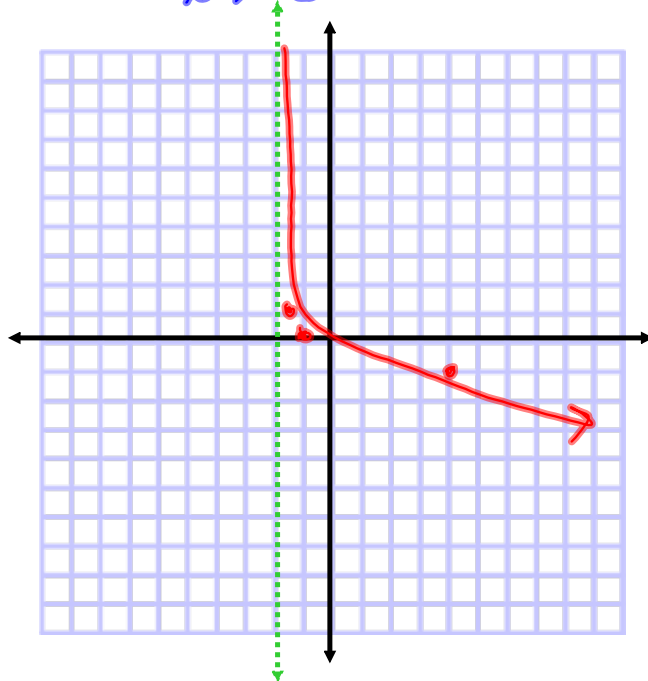
*reflected* (pointing to the negative sign)

*shifted left 2* (pointing to the  $+2$ )

$x > -2$  (under the argument)

HA:  $x = -2$        $6^?$

$x$	
4	-1
-1	0
$-\frac{5}{6}$	1



Graph:  $g(x) = \log_{10}(x-1) + 4$

HA:  $x=1$

