

5.4 Solving Exponential and Logarithmic Functions

$$\log_4 \frac{36}{9} = \log_4 4 = 1$$

$$\log_4 (4 \cdot 9) - \log_4 9$$

$$\underbrace{\log_4 4} + \underbrace{\log_4 9 - \log_4 9}$$

1

$$\log_7 (7^2)^{\frac{1}{3}}$$

$$\log_7 7^{\frac{2}{3}}$$

$$\frac{2}{3}$$

$$\log bx^5 - \log y$$

$$\log b + 5\log x - \log y$$

$$\frac{1}{5} [\log_b 7x] - (\log_b y)$$

$\log_b \frac{\sqrt[5]{7x}}{y}$

Things to remember:

a) $a^x = a^y$, if and only if $x=y$

b) $\log_a x = \log_a y$ if and only if $x=y$

c) $a^{\log_a x} = x$

$$\sqrt[2]{x^2}$$

$$(\sqrt{x})^2$$

d) $\log_a a^x = x$

Solve: $e^x=72$

$$\ln e^x = \ln 72 \quad \left. \begin{array}{l} x \approx \\ 2 - 3 \\ 6 \quad 4 \\ 72 \end{array} \right\}$$

$x = 4.28$

$\ln(72)$
4.276666119

Solve: $4e^{2x} - 3 = 2$

$$4e^{2x} = 5$$

$$e^{2x} = \frac{5}{4}$$

$$\begin{aligned} \ln(5/4) & .2231435513 \\ \text{Ans}/2 & .1115717757 \end{aligned}$$

$$\ln e^{2x} = \ln \frac{5}{4}$$

$$2x = .22$$

$$x = .11$$

Solve: $2(3^{2t-5}) - 4 = 11$

$$2(3^{2t-5}) = 15$$

$$3^{2t-5} = \frac{15}{2}$$

$$\cancel{\log_3 3}^{2t-5} = \underbrace{\log_3 \frac{15}{2}}$$

$$\log(15/2)/\log(3)$$

1.834043767

$$2t-5 = \frac{\log \frac{15}{2}}{\log 3}$$

$$2t-5 = 1.83$$

$$\log(15/2)/\log(3)$$

1.834043767

Ans+5

6.834043767

Ans/2

3.417021884

$$t = 3.42$$

Solve: $e^{2x} - 3e^x + 2 = 0$

$$a^2 - 3a + 2 = 0$$

$$(a-2)(a-1) = 0$$

$$\underbrace{a = e^x}$$

$$\underline{a} = 2 \quad a = 1$$

$$e^x = 2 \quad e^x = 1$$

$$\ln e^x = \ln 2 \quad x = 0$$

$$x = .69$$

$$(e^x)^2$$

Solve: $\ln x = 2$

$$e^{\cancel{x}x} = e^2$$

$$x = e^2$$

e^2

7.389056099

Solve: $\log_3(5x - 1) = \log_3(x + 7)$

$$5x - 1 = x + 7$$

$$x = -2$$

Solve: $5 + 2\ln x = 4$

$$\frac{\ln x^2}{e} = \frac{-1}{e}$$

$$x^2 = e^{-1}$$

$$x = \pm 1$$

extraneous!

$$2 \ln x = -1$$

$$\ln x = -\frac{1}{2}$$

$$e^{\ln x} = e^{-1/2}$$

$$x = 0.61$$

Solve: $2\log_5 3x = 4$

$$\log_5 3x = 2 \rightarrow 5^2 = 3x$$

$$5^{\cancel{\log_5} 3x} = 5^2$$

$$\rightarrow 3x = 25$$

$$x = \frac{25}{3}$$

Solve: $\ln(x-2) + \ln(2x-3) = 2 \ln x$

$$\ln(2x^2 - 7x + 6) = \ln x^2$$

$$2x^2 - 7x + 6 = x^2$$

$$x^2 - 7x + 6 = 0$$

$$(x-6)(x-1) = 0$$

$$x = 6 \text{ or } \cancel{x = 1}$$

You have deposited \$500 in an account that pays %6.75 interest, compounded **continuously**. How long will it take your money to double?

$$A = Pe^{rt}$$

$$1000 = 500 e^{.0675t}$$

$$\frac{\log(4.5)}{\log(6)} \\ .8394415783$$

Solve:

$$6^{x+1} = 4.5^{3x-1}$$

$$\log_b 6^{x+1} = \log_b 4.5^{3x-1}$$

$$x+1 = (3x-1) \log_b 4.5$$

$$\frac{\log 4.5}{\log 6}$$

$$\log 6$$

$$\downarrow x+1 = .84(3x-1)$$

$$\frac{\log(4.5)}{\log(6)} \\ .8394415783$$

Solve:

$$6^{x+1} = 4.5^{3x-1}$$

$$\log 6^{x+1} = \log 4.5^{3x-1}$$

$$.78(x+1) = .69(3x-1)$$

,

$$\underbrace{\ln e^{2x+1}} = 9$$

$$2x+1 = 9$$

$$x = 4$$