

## 5.5 Multiple Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$= 2 \cos^2 u - 1$$

$$= 1 - 2 \sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

solve:

$$2 \cos x + \sin 2x = 0$$

$$\underline{2 \cos x} + (\underline{2 \sin x \cos x}) = 0$$

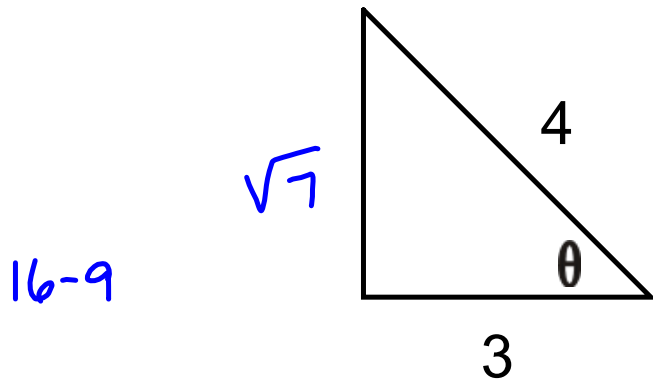
$$2 \cos x (1 + \sin x) = 0$$

$$\cos x = 0$$

$$\frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sin x = -1$$

$$\frac{3\pi}{2}$$



$$\underbrace{\sin 2\theta}_{2 \sin \theta \cos \theta} = \frac{6\sqrt{7}}{16}$$

$$2 \left(\frac{\sqrt{7}}{4}\right) \left(\frac{3}{4}\right) \quad \left(\frac{3\sqrt{7}}{8}\right)$$

$$\left. \begin{array}{l} \sin 2\theta = \frac{6\sqrt{7}}{16} \\ \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} \end{array} \right\}$$

$$\frac{2 \left(\frac{\sqrt{7}}{3}\right)}{1 - \frac{7}{9}} = \frac{2}{\frac{2}{9}}$$

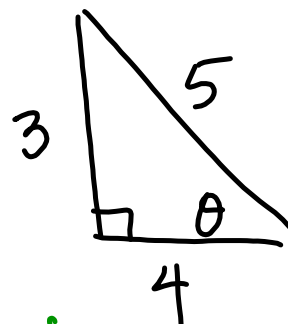
$$\frac{2\sqrt{7}}{3} \cdot \frac{9}{2}$$

$$\left(3\sqrt{7}\right)$$

$$\sin u = \frac{3}{5} \quad 0 < u < \frac{\pi}{2}$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25}$$



$$\tan 2u$$

$$= \frac{2 \tan u}{1 - \tan^2 u} = \frac{2\left(\frac{3}{4}\right)}{1 - \frac{9}{16}} = \frac{\frac{3}{2} \cancel{16}^8}{\frac{7}{16}} = \frac{24}{7}$$

Half Angle Identities:

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

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$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

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$$\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta} \quad \text{or} \quad \frac{\sin \theta}{1 + \cos \theta}$$

$$\sin 105^\circ$$

$$\sin \frac{210^\circ}{2} = \pm \sqrt{\frac{1 - \cos 210^\circ}{2}}$$

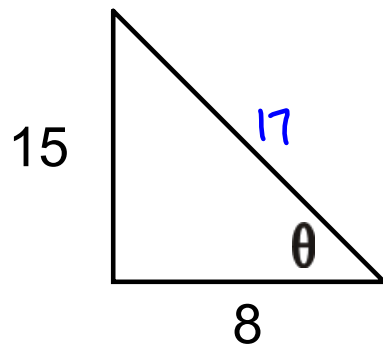
$$= \pm \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}}$$

$$\sqrt{\left(1 + \frac{\sqrt{3}}{2}\right)^{\frac{1}{2}}}$$

$$\sqrt{\frac{\frac{1}{2} + \frac{\sqrt{3}}{4}}{1}}$$

$$\sqrt{\frac{2 + \sqrt{3}}{4}}$$

$$\frac{\sqrt{2 + \sqrt{3}}}{2}$$



$$\cos \frac{\theta}{2}$$

$$\sqrt{\frac{1 + \cos \theta}{2}}$$

$$\sqrt{\frac{1 + \frac{8}{17}}{2}}$$

$$\sqrt{\frac{25}{17} \cdot \frac{1}{2}}$$

$$\sqrt{\frac{25}{34}}$$

$$\frac{5}{\sqrt{34}} = \frac{5\sqrt{34}}{34}$$

$$\sin \frac{\theta}{2}$$

$$\sqrt{\frac{1 - \cos \theta}{2}}$$

$$\sqrt{\frac{1 - \frac{8}{17}}{2}}$$

$$\sqrt{\frac{9}{17} \cdot \frac{1}{2}}$$

$$\sqrt{\frac{9}{34}}$$

$$\frac{3}{\sqrt{34}} = \frac{3\sqrt{34}}{34}$$



