

# Solving Quadratics

## What is a quadratic?

- Polynomial of 2nd degree

- Standard form:

$$ax^2 + bx + c = 0$$

$$\frac{\text{vertex}}{a(x-h)^2 + k}$$

- graph is parabolic

## To Solve Quadratics

- Quadratic Formula
- Factoring
- Graphing
- Square rooting (if  $b = 0$ )
- Completing the square

# Discriminant:

$$b^2 - 4ac$$

$b^2 - 4ac$

positive

two real irrat.  
~~two solutions~~

pos. & perfect

two real rational

$b^2 - 4ac$

zero

one solution

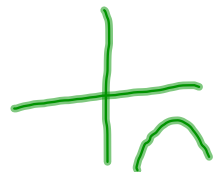


$b^2 - 4ac$

negative

no real solution

2 imag. sol.



Solve by factoring.

$$4x^2 - 10x - 6 = 0$$

$$2(2x^2 - 5x - 3) = 0$$

$$2((2x^2 - 6x) + (x - 3)) = 0$$

$$2[2x(x-3) + 1(x-3)] = 0$$

$$2[(x-3)(2x+1)] = 0$$

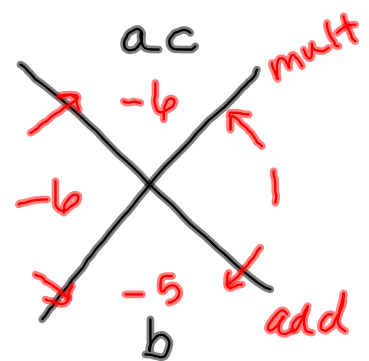
$$2(x-3)(2x+1) = 0$$

$$x-3=0 \quad 2x+1=0$$

$$x=3 \quad x=-\frac{1}{2}$$

$$\frac{b^2 - 4ac}{25 - 4(2)(-3)}$$

$$= 49$$



$$x^2 + 8x = 4$$

$$64 - 4(1)(-4) = 80$$

$$\underbrace{x^2 + 8x + 16}_{(x+4)^2} = 4 + 16$$

$$(x+4)^2 = 20$$

$$x+4 = \pm 2\sqrt{5}$$

$$x = -4 \pm 2\sqrt{5}$$

$$x = \frac{-8 \pm \sqrt{80}}{2}$$

$$x = \frac{-8 \pm 4\sqrt{5}}{2}$$

$$\rightarrow x = -4 \pm 2\sqrt{5}$$

$$9x^2 - 25 = 0$$

Square root method

$$\left. \begin{array}{l} (3x-5)(3x+5)=0 \\ x = \frac{5}{3} \quad x = -\frac{5}{3} \end{array} \right\}$$

$$9x^2 = 25$$

$$x^2 = \frac{25}{9}$$

$$x = \pm \frac{5}{3}$$

$$(x - 3)^2 = 7$$

$$x - 3 = \pm \sqrt{7}$$

$$x = 3 \pm \sqrt{7}$$



## Completing the square:

- force a quadratic to become a perfect square trinomial
- leading coefficient ~~must~~ be 1  
( $a = 1$ )

$$6x = 4 - x^2$$

$$x^2 + 6x - 4 = 0$$

$$\underbrace{x^2 + 6x + 9}_{(x+3)^2} = 4 + 9$$

$$(x+3)^2 = 13$$

$$x+3 = \pm\sqrt{13}$$

$$x = -3 \pm \sqrt{13}$$

$$36 - 4(1)(-4) = 52$$

$$\left(\frac{b}{2}\right)^2$$

$$x^2 + 5x + 2 = 0$$

$$x^2 + 5x + \frac{25}{4} = -2 + \frac{25}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{17}{4}$$

$$x + \frac{5}{2} = \pm \frac{\sqrt{17}}{2}$$

$$x = \frac{-5 \pm \sqrt{17}}{2}$$

$$3x^2 - 18x + 25 = 0$$

$$\begin{aligned} 3x^2 - 18x &= -25 \\ x^2 - 6x + 9 &= -\frac{25}{3} + \frac{27}{3} \\ (x-3)^2 &= \frac{2}{3} \\ x-3 &= \sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{3} \\ x &= 3 \pm \frac{\sqrt{6}}{3} \end{aligned}$$

$$324 - 4(3)(25)$$

$$24$$

$$x = \frac{18 \pm 2\sqrt{6}}{6}$$

$$x = \frac{9 \pm \sqrt{6}}{3}$$

$$9x^2 - 12x = 14$$

$$x^2 - \frac{4}{3}x + \frac{4}{9} = \frac{14}{9} + \frac{4}{9}$$

$$\left(x - \frac{2}{3}\right)^2 = \frac{18}{9}$$

$$x - \frac{2}{3} = \sqrt{2}$$

$$x = \frac{2}{3} \pm \sqrt{2}$$

$$\frac{4}{3} \cdot \frac{1}{2} = \frac{4}{6}$$
$$\frac{2}{3}$$

Solve using any method:

$$3x + 4 = 2x^2 - 7$$

$$9 - 4(2)(-11) = \\ = 97$$

$$2x^2 - 3x - 11$$

$$x = \frac{3 \pm \sqrt{97}}{4}$$

$$x = \frac{3 \pm \sqrt{97}}{4}$$

