Binomial Theorem

What is a binomial?

$(x+y)$

$(x-y)$

$(a+b)$

$2x+3y$

$8a-5b$

$(14j-32f)^8$

a^2+b^2

$$\begin{aligned}
(a+b)^0 &= 1\\
(a+b)^1 &= a+b\\
(a+b)^2 &= a^2+2ab+b^2\\
(a+b)^3 &= a^3+3a^2b+3ab^2+b^3\\
(a+b)^4 &= a^4+4a^3b+6a^2b^2+4ab^3+b^4\\
(a+b)^5 &= a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5
\end{aligned}$$

What do we notice?

- In each expansion there are n+1 terms
- In each expansion, a and b have symmetrical roles
- The sume of the powers of each term is n.
- The coefficients increase and then decrease in a symmetric pattern.

Finding binomial coefficients

$$
\begin{array}{c}
1\\
1 \quad 1\\
1 \quad 2 \quad 1\\
1 \quad 3 \quad 3 \quad 1\\
1 \quad 4 \quad 6 \quad 4 \quad 1\\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1\\
1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1\\
1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1
\end{array}
$$

$(x+y)^8$

5th term

$$ {}_{n}C_{r} = \frac{\overset{8}{n}!}{(n-\underset{4}{r})!\, r!} = \frac{8!}{(\cancel{4!})(4!)} = \frac{\overset{2}{\cancel{8}}\cdot 7\cdot \cancel{6}\cdot 5}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}} = 70 $$

Power term-1

Find the coefficient of the 6th term in the expansion of $(x + y)^{11}$

The Binomial Theorem:

$$(x + y)^n = x^n + nx^{n-1}y + \ldots + {}_nC_r\, x^{n-r} y^r + \ldots + nxy^{n-1} + y^n$$

Expand:

$$(x + y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$$

Another way to find coefficents:

$$(a+b)^0 = 1$$
$$(a+b)^1 = a+b$$
$$(a+b)^2 = a^2+2ab+b^2$$
$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$$
$$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$$
$$(a+b)^5 = a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$$

Pascal's Triangle:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

Expand:

$(x + y)^8 =$

$x^8 + 8x^7y + 28x^6y^2 + 56x^5y^3 + 70x^4y^4 + 56x^3y^5 + 28x^2y^6 + 8xy^7 + y^8$

$(2x - 3)^4 = (2x)^4 + 4(2x)^3(-3) + 6(2x)^2(-3)^2 + 4(2x)(-3)^3 + (-3)^4$

$4 \cdot 2^3 \cdot -3 \quad 6 \cdot 4 \cdot 9 \quad 4 \cdot 2 \cdot -27$

$16x^4 - 96x^3 + 216x^2 - 216x + 81$

$(x^2 + 4)^3 = (x^2)^3 + (x^2)^2(4) + (x^2)(4)^2 + (4)^3$

$x^6 + 4x^4 + 16x^2 + 64$

Pascal's

Find the sixth term of $(a + 2b)^8$

1 8 28 56 70 56 28 8 1
1 2 3 4 5 6

$(x + y)^n = x^n + nx^{n-1}y + \ldots + {}_nC_r\, x^{n-r} y^r + \ldots + nxy^{n-1} + y^n$

$$56(a)^3(2b)^5 = \boxed{1792a^3b^5}$$

$56 \cdot 32$

Find the coefficient of the term a^6b^5 in the expansion of $(3x - 2b)^{11}$

Pascals

6th term

1 11 55 165 330 462 330 165 55 11 1

11 10 9 8 7 6
0 1 2 3 4 5

$$= 462(3x)^6(-2b)^5$$

$462 \cdot 729 \cdot -32$

$= \boxed{10777536x^6b^5}$

$\boxed{\text{OR}}$

$$\underbrace{C}_{11\quad 5}\,(3x)^6(-2b)^5$$

$$\frac{11!}{6!5!}\,(3x)^6(-2b)^5$$

729 -32

$$\frac{11\cdot \cancel{10}^{\,3}\cdot \cancel{9}\cdot \cancel{8}^{\,2}\cdot 7}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = 462(729)(-32)$$

$$= \boxed{10777536x^6b^5}$$